Conformal Geometry Based Supine and Prone Colon Registration

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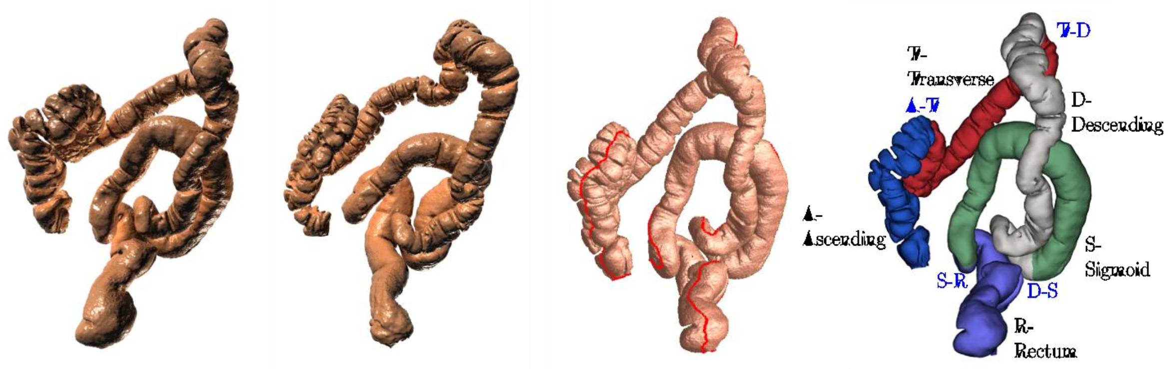
Abstract. In virtual colonoscopy, CT scans are typically acquired with the patient in both supine and prone positions. The registration of these two scans is desirable so that the physician can clarify situations or confirm polyp findings at a location in one scan with the same location in the other, thereby improving polyp detection rates and reducing false positives. However, this supine-prone registration is challenging because of the substantial distortions in the colon shape due to the patient's position shifting. We present an efficient algorithm and framework for performing this registration through the use of conformal geometry to guarantee the registration is a diffeomorphism. The colon surface is conformally flattened to a rectangle using holomorphic differentials. The flattened domains of supine and prone are aligned by the harmonic map with feature correspondence constraints. We demonstrate the efficiency and efficacy of our method by measuring the distance between features on the registered colons.

Keywords: virtual colonoscopy, supine-prone registration, conformal geometry

1 Introduction

Virtual colonoscopy (VC) techniques have been developed as viable non-invasive alternatives to optical colonoscopy (OC) for screening purposes [7, 11]. For a VC procedure, computed tomography (CT) scans of the abdomen are commonly acquired with the patient in both the supine (facing up) and prone (facing down) positions. From these scans, the colon wall can be extracted as in Fig. 1 (a-b) and presented to the VC reader in various ways, including as a volume rendered endoluminal view, mimicking the endoscopic view of an OC, from both supine and prone data.

The use of computer-aided detection (CAD) of colonic polyps [9, 15] can help to reduce the necessary reading and interpretation time of the user and can act as a second reader to improve detection rates of VC. Though various CAD methods can achieve different accuracies, a common problem among them is the presence of false positives. A reduction of these false positives would help the user to focus on true suspicious areas and not waste time on unimportant regions. Throughout the development of VC, the registration of the supine and prone scans has remained a constant and challenging problem [1, 3, 13]. Being able to register these two scans is useful for both a routine VC system and for a CAD system. In the case of a VC system, providing the user the ability to jump from one area in one scan to the same area in the other scan would allow for the easy comparison of these areas when something might be unclear in one of the scans, or for confirming a finding. For a CAD system, a proper registration could help achieve greater accuracy while at the same time reducing false positive results.



(a) supine (b) prone (c) taenia coli (d) flexures

Fig. 1. Feature extraction of supine-prone colons.

In this paper, we present a method of supine-prone registration based on conformal geometry. Conformal colon flattening has been introduced as an enhancement for VC navigation [8] and utilized successfully for CAD [9]. According to conformal geometry theory, there exists an angle preserving map which flattens the colon surface onto a planar rectangle. This mapping minimizes the total stretching energy. Because of the local shape preserving property, it offers an effective way to visualize the entire colon surface, and exposes all of the geometric structures hidden in the original shape embedded in 3D.

The non-rigid elastic deformation between supine and prone colons poses a great challenge for shape registration. In this work, we locate and match the anatomical feature curves (flexures) and internal feature points on the conformally flattened supine and prone surfaces, and compute a harmonic map with these feature constraints. Then, we obtain a diffeomorphism between the supine and prone colons. Our registration method performs better than other existing centerline methods. To the best of our knowledge, it is the first work to apply geometric mapping for supine-prone colon registration by converting the 3D registration problem to a 2D image matching problem.

2 Algorithm Overview

The computational details include the following steps:

1. Conformal mapping: A flat rectangular conformal mapping is computed for the colon segments of the supine and prone surfaces using holomorphic differentials (see Fig. 2 (a-b)). As the preprocessing step, we extract the anatomical landmarks (taenia coli and flexures) and use them to decompose the colon surfaces to segments and slice them open, as shown in Fig. 1 (c-d).

2. Registration: The supine-prone colon registration is performed using a harmonic map with the feature correspondence constraints. The registration process and results are illustrated in Fig. 2. We extract the internal features and computed the constraints on the conformal mapping images with color encoded mean curvature, using the well-known graph cut segmentation and the graph matching method [2].

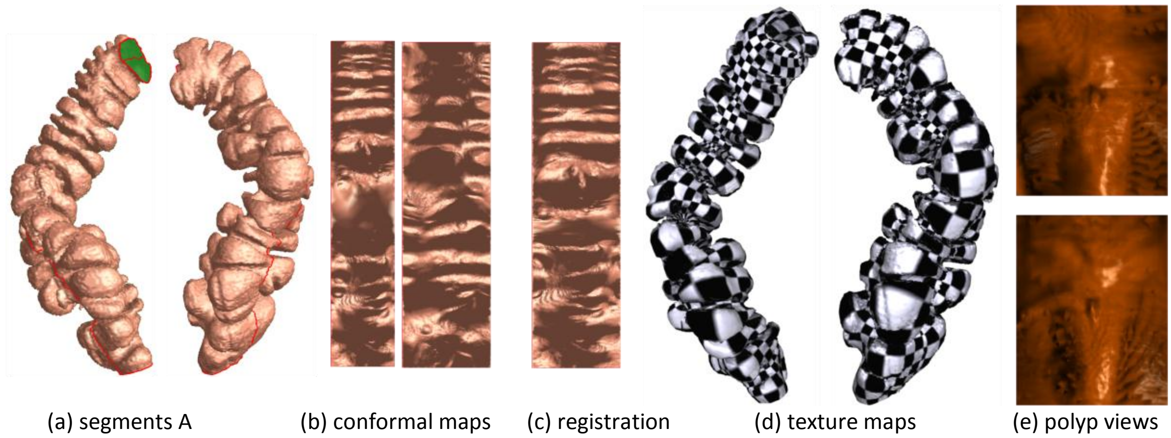


Fig. 2. Registration for segments *A* of supine (left) and prone (right) colons. The prone segment is the reference for registration; (b) the conformal maps of segments *A* in (a); (c) the result of supine segment registered to prone segment; (d) the checker-board texture mapping for consistency visualization; (e) the volume rendering results for the consistent views of a polyp.

3 Conformal Mapping

This section briefly explains the algorithm for computing the flattened colon by conformal mapping method. The colon surface is a topological cylinder with two boundaries. In practice, all surfaces are approximated by piecewise linear polygonal meshes. Here, we model the colon surfaces as triangular meshes.

Given such a mesh *Q*, in order to find a conformal mapping $\phi : Q \to \mathbf{C}$, which maps *Q* to a planar rectangle, we compute two harmonic functions $f_1, f_2 : Q \to \mathbf{R}$ with the Dirichlet boundary conditions [8]. The desired holomorphic 1-form is defined as $\omega = \nabla f_1 + \sqrt{-1}\lambda \nabla f_2$, where ∇f_1 is a closed harmonic 1-form, ∇f_2 is an exact harmonic 1-form, and λ is a scalar, such that $* \nabla f_1 = \lambda \nabla f_2$. The induced conformal mapping $\phi : Q \to \mathbf{C}$ is given by

$$\phi(p) = \int_q^p \omega,$$

where q is the base point and the path from q to p is arbitrarily chosen. Then, the surface is conformally mapped to a planar domain. By tracing the straight line perpendicular to the two boundaries, we obtain a rectangular fundamental domain.

As shown in Fig. 2, the supine and prone colon segments Q_1, Q_2 in (a) are the topological cylinders with two boundaries, cut from the flexures. We slice it open to a topological disk along the taenia coli, which connects the two boundaries. Then, we compute the harmonic functions and holomorphic differentials to conformally map the surfaces to rectangles in (b). The conformal maps are denoted as ϕ_1 and ϕ_2, respectively.

For the theoretical background and computational details about holomorphic differentials, we refer readers to previous works [6, 17, 18, 19].

4 Registration by Harmonic Map

This section explains the details of computing the harmonic mapping between supine and prone colon surfaces for registration purpose. Suppose the feature points are $\{p_0, p_1, \ldots, p_n\}$

on the supine surface and $\{q_0, q_1, \ldots, q_n\}$ on the prone surface, such that p_k corresponds to q_k. In order to enforce the alignments among these internal features, first we use an affine map $\eta : \mathbf{R}^2 \to \mathbf{R}^2$, which maps the rectangle of the supine to that of the prone. Then we compute two harmonic functions, $\mathbf{h} = (h_1, h_2)$, such that

$$\mathrm{h}(p_k) = \phi_2(q_k) - \eta \circ \phi_1(p_k), 0 \le k \le n,$$

and furthermore

$$\Delta h_1 = 0, h_1 |_{\gamma_2 \cup \gamma_4} = 0, \frac{\partial h_1}{\partial \mathrm{n}} |_{\gamma_1 \cup \gamma_3} = 0,$$

and

$$\Delta h_2 = 0, h_2 |_{\gamma_1 \cup \gamma_3} = 0, \frac{\partial h_2}{\partial \mathrm{n}} |_{\gamma_2 \cup \gamma_4} = 0.$$

Then, the final registration map from supine to prone $\Phi : S_1 \to S_2$ is given by

$$\Phi := \phi_2^{-1} \circ (\mathrm{h} + \eta \circ \phi_1).$$

Figure 2 shows a registration example of segments A (a) based on such harmonic maps. The prone segment is specified as the reference for registration. Compared to the initial conformal mappings in (b), it is clear that the registration map of the supine segment (c) achieves greater accuracy when using the internal features. The geometry registration is visualized by the consistent checker-board texture mapping in (d). With the motivation of enhancing the polyp detection accuracy, we locate the possible polyps in supine, then jump to the same location in prone and obtain the consistent view through the registration result. In this way, the physician can make more accurate decisions by the double confirmation. The volume rendering in (e) helps demonstrate exact appearance.

5 Experimental Results

We validate our algorithms using real VC colon data from the publicly available National Institute of Biomedical Imaging and Bioengineering (NIBIB) Image and Clinical Data Repository provided by the National Institute of Health (NIH). We perform electronic colon cleansing incorporating the partial volume effect [16], segmentation with topological simplification [9], and reconstruction of the colon surface via surface nets [5] on the original CT images in a pre-processing step. In this paper, the colon surface is modeled as a topological cylinder and discretely represented by a triangular mesh.

We evaluate our registration results by an objective analytic evaluation, whereby distances between corresponding points on the registered colons are calculated. We compute the 3D distance error in millimeters. For the two corresponding points p0 and q0 in R2, we know their locations r0 and s0 in R3. If we take the supine surface (containing p0) as the truth and wish to measure the registration error on the prone surface (containing q0), we can identify the point p1 = (uq, vq) in R2 on the supine surface and similarly its location r1, in R3. The distance error is then given to be | r1 - r0|. Previous works have most often focused on centerline alignment. The ground truth for colon deformation is the whole surface deformation; the centerline only conveys very limited information. Since our method uses this surface instead of the centerline, it is expected that we achieve better results than the cruder centerline methods. Our experiments on 6 pairs of supine-prone colons obtained an average R3 distance error of 7.85mm in terms of (a) the feature points and (b) polyps evaluation. Table 1 shows that our method produces a registration with significantly smaller distance error between corresponding points than other centerline methods with distance error, and similar results to another method based on the registration of haustral folds [4]. Unlike all the other methods, our algorithm provides a one-to-one and onto mapping between the two colon surfaces, allowing for precise localization of corresponding positions whether on a haustral fold or not.

Table 1. Comparison of average millimeter distance error between existing methods.

Methods	**Distance Error**
Our Conformal Geometry Based Method	**7.85mm**
Haustral fold registration [4]	5.03 mm
Centerline registration + statistical analysis [12]	12.66mm
Linear stretching / shrinking of centerline [1]	13.20mm
Centerline feature matching + lumen deformation [14]	13.77mm
Centerline point correlation [3]	20.00mm
Taenia coli correlation [10]	23.33mm

6 Conclusion

Shape registration is very fundamental for shape analysis problems, especially for illness and abnormality detection in medical applications. We introduce an efficient framework for the registration of supine and prone colons, through the use of conformal geometry, to improve the accuracy of polyp detection. Experimental results demonstrate that our registration method performs better than other existing methods. To the best of our knowledge, this is the first work to use the geometric mapping method for the supine and prone colon surface registration problem. In the future, we will investigate the registration for volume data.

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References

1. B. Acar, S. Napel, D. S. Paik, P. Li, J. Yee, R. B. Jeffrey, Jr., and C. Beaulieu. Medial axis registration of supine and prone CT colonography data. Proc. of Engineering in Medicine and Biology Society (EMBS), 2433-2436, Oct. 2001.
2. Y. Boykov and V. Kolmogorov. An experimental comparison of min-cut/maxflow algorithms for energy minimization in vision. IEEE Transactions on Pattern Analysis and Machine Intelligence, 26(9):1124-1137, Sept. 2004.
3. H. de Vries, R. Truyen, J. van der Peijl, J. Florie, R. E. van Gelder, F. Gerritsen, and J. Stoker. Feasibility of automated matching of supine and prone CT-colonography examinations. British Journal of Radiology, 79:740-744, Sept. 2006.
4. Fukano, M. Oda, T. Kitasaka, Y. Suenaga, T. Takayama, H. Takabatake, M. Mori, H. Natori, S. Nawano, and K. Mori. Haustral fold registration in CT colonography and its application to registration of virtual stretched view of the colon. Proc. SPIE Medical Imaging vol. 7624, 1–11, 2010.
5. S. F. F. Gibson. Constrained elastic surface nets: Generating smooth surfaces from binary segmented data. Proc. of MICCAI, pages 888-898, 1998.

6. X. Gu and S.-T. Yau. Global conformal parameterization. Symposium on Geometry Processing, pages 127-137, 2003.
7. L. Hong, S. Muraki, A. Kaufman, D. Bartz, and T. He. Virtual voyage: Interactive navigation in the human colon. Proc. of SIGGRAPH, 27-34, 1997.
8. W. Hong, X. Gu, F. Qiu, M. Jin, and A. Kaufman. Conformal virtual colon flattening. In ACM Symposium on Solid and Physical Modeling, 85-93, 2006.
9. W. Hong, F. Qiu, and A. Kaufman. A pipeline for computer aided polyp detection. IEEE Transactions on Visualization and Computer Graphics, 12(5):861-868, Sept. 2006.
10. Huang, D. Roy, M. Franaszek, and R. M. Summers. Teniae coli guided navigation and registration for virtual colonoscopy. Proc. of IEEE Visualization, 279-285, Oct. 2005.
11. D. Johnson and A. H. Dachman. CT colography: The next colon screening examination. Radiology, 216(2):331-341, 2000.
12. P. Li, S. Napel, B. Acar, D. S. Paik, R. B. Jeffrey, Jr., and C. F. Beaulieu. Registration of central paths and colonic polyps between supine and prone scans in computed tomography colonography: Pilot study. Medical Physics, 31(10):2912-23, Oct. 2004.
13. J. Näppi, A. Okamura, H. Frimmel, A. Dachman, and H. Yoshida. Region-based supine-prone correspondence of false-positive CAD polyp candidates in CT colonography. Academic Radiology, 12:695-707, 2005.
14. J. W. Suh and C. L. Wyatt. Deformable registration of supine and prone colons for computed tomographic colonography. Journal of Computer Assisted Tomography, 33(6):902-911, 2009.
15. V. van Ravesteijn, L. Zhao, C. Botha, F. Post, F. Vos, and L. van Vliet. Combining mesh volume and streamline representations for polyp detection in CT colonography. Proc. of International Symposium on Biomedical Imaging (ISBI), pages 907-910, 2009.
16. Z. Wang, Z. Liang, L. Li, B. Li, D. Eremina, and H. Lu. An improved electronic colon cleansing method for detection of colonic polyps by virtual colonoscopy. IEEE Transactions on Biomedical Engineering, 53(8):1635-46, Aug. 2006.
17. W. Zeng, L. M. Lui, X. Gu, and S.-T. Yau. Shape analysis by conformal modules. Methods and Applications of Analysis, 15(4):539-556, 2009.
18. W. Zeng, D. Samaras, and X. Gu. Ricci flow for 3D shape analysis. IEEE Transactions on Pattern Analysis and Machine Intelligence, 32(4): 662-677, 2010.
19. W. Zeng, Y. Zeng, Y. Wang, X. Yin, X. Gu, and D. Samaras. 3D non-rigid surface matching and registration based on holomorphic differentials. The 10th European Conference on Computer Vision, France, Oct. 2008.